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2.26

$$U |S_z; +\rangle = |S_x; +\rangle \Rightarrow \langle S_z; + | U |S_z; +\rangle = \langle S_z; + | S_x; +\rangle$$

$$U |S_z; -\rangle = |S_x; -\rangle \quad \langle S_z; - | U |S_z; +\rangle = \langle S_z; - | S_x; +\rangle$$

$$U = 1 \quad \langle S_z; + | U |S_z; -\rangle = \langle S_z; - | S_x; -\rangle$$

$$\langle S_z; - | U |S_z; -\rangle = \langle S_z; - | S_x; -\rangle$$

$$|S_x; \pm\rangle = \frac{1}{\sqrt{2}} |S_z; +\rangle \pm \frac{1}{\sqrt{2}} |S_z; -\rangle$$

$$\langle S_z; + | S_x; \pm\rangle = \frac{1}{\sqrt{2}}, \quad \langle S_z; - | S_x; \pm\rangle = \pm \frac{1}{\sqrt{2}}$$

$$U = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$U = |S_x; +\rangle \langle S_z; +| + |S_x; -\rangle \langle S_z; -|$$

$$= \frac{1}{\sqrt{2}} (|+\rangle \langle +| + |-\rangle \langle +| + |+\rangle \langle -| + |-\rangle \langle -|)$$

Then

$$\langle + | U | + \rangle = \frac{1}{\sqrt{2}}, \quad \langle + | U | - \rangle = \frac{1}{\sqrt{2}}, \quad \langle - | U | + \rangle = \frac{1}{\sqrt{2}}, \quad \langle - | U | - \rangle = -\frac{1}{\sqrt{2}}$$

so

$$U = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$