

1.19

$$S_x = \frac{\hbar}{2} (|+\rangle\langle -| + |-\rangle\langle +|), \quad S_y = \frac{i\hbar}{2} (|-\rangle\langle -| - |+\rangle\langle +|)$$

$$S_x^2 = \left(\frac{\hbar}{2}\right)^2 (|+\rangle\langle +| + |-\rangle\langle -|), \quad S_y^2 = -\left(\frac{\hbar}{2}\right)^2 (|-\rangle\langle -| - |+\rangle\langle +|)$$

$$[S_x, S_y] = i\hbar S_z$$

$$\langle S_x \rangle = \langle + | S_x | + \rangle = 0 \quad \langle S_y \rangle = 0$$

$$\langle S_x^2 \rangle = \left(\frac{\hbar}{2}\right)^2, \quad \langle S_y^2 \rangle = \left(\frac{\hbar}{2}\right)^2$$

$$\langle (S_x)^2 \rangle = \langle (S_y)^2 \rangle = \left(\frac{\hbar}{2}\right)^2$$

$$\langle [S_x, S_y] \rangle = \langle i\hbar S_z \rangle = i\hbar \left[\frac{\hbar}{2} (1 - 1) \right] = 0$$

So

$$\left(\frac{\hbar}{2}\right)^2 \left(\frac{\hbar}{2}\right)^2 \geq 0$$

for the S_x^+ state, $\langle (S_x)^2 \rangle = 0$

$$\text{So } 0 \cdot \langle (S_y)^2 \rangle \geq \frac{1}{4} |\langle [S_x, S_y] \rangle|^2 = 0$$

i.e. $0 \geq 0$, trivial. There is no uncertainty in this state

Since the spin is measured to be in the S_x^+ direction

measuring the S_y state is merely a projection of the S_x^+ state onto the S_y basis.